EASY COROLLARIES IN NON-EUCLIDEAN GEOMETRY.

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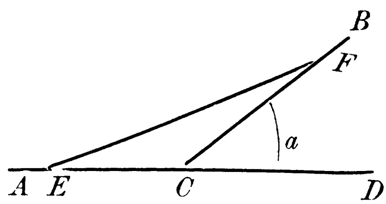
In that sort of Non-Euclidean geometry usually named after Lobatschewsky, it is readily demonstrated that the angle-sum of a rectilineal triangle is a variable directly connected with the size of the triangle, just as is the case in spherics. This proof in a very elementary form is given by John Bolyai. His section 41 is, "Equivalent triangles have their angle-sums equal." Then as an easy corollary, section 42, "Triangles are to each other as the supplements of their angle-sums."

From this we get, at once, the corollary, that in Lobatschewsky's geometry there may be a triangle whose angle-sum differs from a straight angle by less than any given finite angle however small.

For a single angle ACB can always be drawn less than a straight angle and in such manner as that it shall differ from a straight angle by as small an angle, a, as any given finite angle however small. Then drawing a straight line from any point E on the arm AC to any point F on the arm BC we shall have a triangle ECF, the supplement of whose angle-sum is less than a.



Hence the further corollary, that we can always draw a triangle less in size than any triangle whose angle-sum is less than a straight angle by the given finite angle a.

BIBLIOGRAPHY OF THE HISTORY OF GEOMETRY; ALSO A LIST OF MATHEMATICAL PERIODICALS.

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The following list was prepared in the belief that it would be of interest and value to those who are making a study of the history and developement of Geometry. The list does not pretend to be complete, yet it is thought that the principal English, French, and German works are included. The list of American Periodicals is thought to be complete. The date given in connection with each periodical is the date of first issue. In the case of periodicals that have been discontinued, the date of discontinuation, when possible, is given. The list has been made by a careful study of the references in the leading histories of Mathematics. The place of publication of a number of the periodicals has not been determined with certainty; these are left blank in the list. Corrections and additions will be gladly received by the writer.

ENGLISH.

Adams, George	*Geometrical and Graphical Essays*,	1797.
Allman, G. J.	*Greek Geometry from Thales to Euclid*,	1889.
Barrow, I.	*Translation of Euclid*,—15 books,	1660.
Ball, R. W.	*A Short History of Mathematics*,	1888.
Ball, R. W.	*History of the Study of Mathematics at Cambridge*,	1889.
Ball, R. W.	*Mathematical Recreations*,	1889.
Bossut, C.	*A General History of Mathematics*,	1803.
Barlow, Peter	*A Mathematical and Philosophical Dictionary*,	1814.
Burrow, R.	*Translation of the Geometrical Treatise of Apollonius*,	1779.
Cajorie, F.	*A History of the Study and Teaching of Mathematics in the United States*,	1890.
Cajorie, F.	*A History of Mathematics*,	1894.
Dodgson, C. L.	*Euclid and his Modern Rivals*,	1879.
Donn, Benj.	*The Geometrician; Essays on Plane Geometry*,	1778.
DeMorgan, A.	*Euklides, in Smith's Dictionary of Greek and Roman Biography*,	1849.
DeMorgan, A.	*Budget of Parodoxes*,	1872.
Davies, T. S.	*Geometry and Geometers*,	1850.
Gow, J.	*History of Greek Mathematics*,	1884.
Holyoake, G. J.	*Mathematics no Mystery, or the Beauties and Uses of Euclid*,	n. d.
Hutton, Chas.	*A Mathematical Dictionary*,—2 Vols.,	1815.
Moxon, Jas.	*Mathematicks Made Easie;or a Mathematical Dictionary*,	1692.
Newman, F. W.	*Difficulties of Elementary Geometry*,	1841.
Paman, Roger	*The Harmony of the Ancient and Modern Geometry Asserted*,	1745.
Playfair, John	*Progress of Mathematical and Physical Science since the Revival of Letters in Europe*,—2 Vols.,	1816.
Pirie, G.	*Short Account of the Principal Geometrical Methods of Approximating to the Value of* π,	1877.
Plucker, J.	*On a New Geometry of Space*,	1865.
Shanks, W.	*Contributions to Mathematics*, (*Squaring the Circle*),	1853.
Stone, E.	*A New Mathematical Dictionary*,	1726.
Smith, Jas.	*The Quadrature of the Circle.* Correspondence between an eminent mathematician (A. DeMorgan) and J. Smith,	1861.
Thompson, T. P.	*Geometry Without Axioms*,	1833.
Taylor, Thos.	*Proclus' Commentaries on Euclid*, 2 Vols.,	1792.
Taylor, Chas.	*An Introduction to the Ancient and Modern Geometry of Conics.*	1881.
Whewell, W.	*History of the Inductive Sciences*,	1840.
Wilkins, John	*Mathematicall Magick, or the Wonders that May be Performed by Mechanical Geometry*,	1648.

FRENCH.

Chaignet, A. E. *Pythagore et la Philosophie Pythagoricienne*, 1874.
Charles, M. *Rapport sur les Progres de la Geometrie*, 1870.
Charles, M. *Apercu historique sur l'origine et le developpement des Methodes en Geometrie*, 1875.
D'Alembert, Bossut,&c. *Dictionnaire Encyclopedique des Mathematiques*, 4 Vols., 1789.
Hoefer, T. *Histoire des Mathematiques depuis leurs origines jusqu, an commencement du dix-neuvieme siecle*, 1874.
Libri, Guil *Histoire des Sciences Mathematiques en Italie*, 4 Vols, 1838.
Marie, M. *Histoire des Sciences Mathematiques et Physiques*, 12 Vols., 1888.
Montucla, J. S. *Histoire des Mathematiques*, 4 Vols. 1802.
Montucla, J. S. *Histoire des Recherches sur la Quadrature du Cercle*, 1831.
Montferrier, A. S. de *Dictionnaire des Sciences Mathematiques pures et appliques*, 2 Vols. 1836.
Mansion, P. *Histoire des Mathematiques dans l'antiquite et an moyen age*, 1875.
Ozanam, *Dictionnaire Mathematique*, 1691.
Ozanam, Jas. *Recreations Mathematiques et Physiques*, 4 Vols., 1778.
Quetelet, L. A. J. *Histoire des Sciences Mathematiques et Physiques chez les Belges*, 1866.
Saverin, Alex. *Dictionnaire universel de Mathematique*, 2 Vols., 1753.
Saverin, Alex. *Histoire des Sciences exactes*, 1776.
Sonnet, H. *Dictionnaire des Mathematsiques*, 1867.
Sedillot, L. A. *Materiaux pour servir a l'histoire des Sciences Mathematiques chez les Grecs et les Orientaux*, 1849.
Tannery, P. *Geometrie Grecque*, 1887.
Tannery, P. *Sur les Solutions du Probleme de Delos par Archytas et par Eudoxe.*
Tannery, P. *Histoire general de la geometrie elementaire*, 1887.
Tannery, P. *Histoire de la Science Hellene de Thales a Empedocle.* 1887.

GERMAN.

Arneth, A. *Die Geschichte der reinen Mathematik*, 1851.
Bretschneider, C. A. *Die Geometrie und die Geometee vor Euklides*, 1876.
Breitschwert, L. E. von. *Johann Kepler's Leben und Werken*, 1831.
Busch, J. G. *Encyclopaedie der Mathematischn Wissenschaften*, 1795.
Baltzer, R. *Mathematisch-historisch Bemerkungen*, 1865.
Beier, O. *Der Mathematik im Unterrichte*, 1879.
Cantor, M. *Vorlesungen uber Geschichte der Mathematik*, 1880.
Chasles, M. *Geschichte der Geometrie*, 1839.
Eisenbohr, A. *Ein Mathematischn Handbuch der alten Ægypter*, 1877.
Encke, J. F. *Geschichte Mathematik und Astronomie*, 3 Vols., 1889.
Friedlein, G. *Die Geometrie der Boethins und die indischen Ziffern*, 1861.
Gunther, M. S. *Geschichte der Mathematischen Wissensehaften*, 1876.
Gruppe, *Ueber die Fragmente der Archytas und der aelteren Pythagoner*, 1840.

Gunthee, M. S.	*Die Geschichte Mathematischen Unterichts*,	1877.
Geshardt, C. J.	*Geschichte der Mathematik in Deutchland*,	1877.
Gherardi,	*Die Geschichte der Mathematischen Facultat in Bologna.*	
Heiberg,	*Studiren uber Euklid*,	1882.
Hankel, H.	*Geschichte der Mathematik*,	1874.
Hoffmann, J. J.	*Der Pythagorische Lersatz*,	1821.
Kunsberg, Hans.	*Der Astronom., Mathemat., a Geograph. Eudoxos von Knidos*,	1888.
Kæstner, A. G.	*Geschichte der Mathematik*, 4 Vols.,	1806.
Klugel,	*Mathematische- Worterbuch*, 7 Vols.,	1836.
Poppe, J. H.	*Geschichte der Mathematik*,	1828.
Reye, Th.	*Die Synthetische Geometrie in Altertheum und in der Neuzeit*,	1886.
Suter, H.	*Geschichte der Mathematischen Wissenschaften*,	1873.
Suter, H.	*Die Mathematik auf den Universitaten des Mittelalter*,	1887.
Schmidt, C. P.	*Die Fragmente des Mathematikers Menaechmus*,	1884.
Spottiswoode, W.	*Die Mathematik in ihren Beziehung zu den anderen Wissenschaften*,	1879.
Sturm, R.	*Die Entwickelung der Geometrie*,	1886.
Von Kremer, A.	*Kultergeschichte des Orients unter den Chalifen*,	1877.
Weyr, E.	*Die Geometrie der alten Ægyptet*,	1884.
Weissenborn, H.	*Keuntruss des Mathematik des Mittelalters*,	1888.
Wockel, H.	*Geometrie der Alten*,	1880.

PERIODICALS, AMERICAN.

Des Moines,	*Analyst, The*, 10 Vols.,	1874-84.
New York,	*Analyst, The or Mathematical Museum*, 5 Nos.,	1808.
Charlottesville,	*Annals of Mathematics*,	1884.
Baltimore,	*American Journal of Mathematics, The*	1878.
Kidder,	AMERICAN MATHEMATICAL MONTHLY, THE	1894.
	American Association Proceedings,	1851.
Philadelphia,	*American Philosophical Society Proceedings*,	
New York,	*Bulletin of the N. Y. Mathematical Society*,	1891.
Cambridge,	*Cambridge Miscellany of Math., Physics, and Astronomy*, 4 Nos.,	1842.
Boston,	*Enquirer, The, or Mathematical and Philosophical Repository*, 3 Vols.,	1811-14.
New York,	*Ladies' and Gentlemens' Diary*; 3 Nos.,	1820-22.
Washington,	*Mathematical Magazine*, A. Martin, 13 Nos.,	1882-90.
Cambridge,	*Mathematical Monthly, Runkles*, 3 Vols.,	1859-61.
New York,	*Mathematical Correspondent, The* 1 Vol.,	1804.
New York,	*Mathematical Diary, The* 13 Nos.,	1825-32.
Flushing,	*Mathematical Miscellany, The* 8 Nos.,	1836-39.
New York,	*Mathematical Companion, The* 4 Nos.,	1828-32.
Ada,	*Mathematical Messenger, The*	1884.
Washington,	*Mathematical Visitor*, A. Martin,	1877.
Perth Amboy,	*Scientific Journal, The* 9 Nos.,	1818-19

Worcester,	*Schoolmaster, The* 1 Vol.,	1832.
Rolla,	*Scientiae Bacalaureus*, 1 Vol.,	1890.

PERIODICALS, FOREIGN.

Rome,	*Annali di Matimatica pure ed applicata*,	1873.
Paris,	*Annales scientifiques de l'Ecole Normale Lupesieure*,	1864.
Paris,	*Annales de Mathematiques pures et appliquees*,	1810-31.
Stockholm, Berlin, Paris,	*Acta Mathématica*,	1882.
Paris,	*Bulletin de la Societe Philomthique*,	1788.
Paris,	*Bulletin de la Societe Mathematique de France*,	1873.
Paris,	*Bulletin des Sciences Mathematiques*, Series I and II,	1824.
Cambridge,	*Cambridge Philosophical Transactions*,	1842.
Cambridge,	*Cambridge Mathematical Journal*, 4 Vols.,	1839-45.
Cambridge,	*Cambridge and Dublin Mathematical Journal*, 9 Vols.,	1846-54.
Berlin,	*Crelle's Journal fur die reinen und angewande Mathematik*,	1826.
	Correspondance sur l'Ecole Polytechnique,	1811.
Bruxelles,	*Correspondance Mathematique et Physique*,	1825-39.
Paris,	*Comptes rendus de l'Academie des sciences*,	1832.
London,	*Educational Times*,	1835.
	Enigmatical Entertainer and Mathematical Repository, 4 Nos.,	1827-30.
Liverpool,	*Enquirer, The or Philosophical and Mathematical Repository*,	1825.
Liverpool,	*Geometrical Amusements*,	1821.
	Gentlemen's Diary,	1741-1840.
	Gentlemans' Mathematical Companion, 19 Vols.,	1798-1827
Berlin,	*Jahrbuch uber die Fortschritte der Mathematik*,	1868.
Paris,	*Journal de Mathematiques*, Series 1, 2, 3, and 4,	1836.
	Journal de Mathematiques Elementaire Vinberts.	
	Journal de Mathematiques Specielles et Elementaire.	
	Journal des Sarants.	
Paris,	*Journal de l'Ecole Polytechnique*,	1794.
	Journal l' Sustitut,	1833.
	Leed's Correspondent,	1815-23.
London,	*London Mathematical Society Proceedings*,	1865.
Liverpool,	*Liverpool Apollonius, The* 2 Parts,	1823-24.
	Ladies' and Gentlemen's Diary,	1841-69.
	Ladies' Diary, The	1723-1840.
	Mathematical Magazine, Mitchell and Moss 1 Vol.,	1761.
Leipzig,	*Mathematische Annalen*,	1869.
	Mathematician, The 6 Vols.,	1745-51.
	Mathematician, The 13 Vols.,	1843-56,
	Mathematical Repository, The	1806-35.
	Mathematical Exercises, 6 Nos.,	1750-53.
Paris,	*Memoires des Sarants etrangers*,	1833.
Berlin,	*Memoires de l'Academie de Berlin.*	

Paris,	*Memoires de l'Academie des Sciences.*	
London,	*Messenger of Mathematics*, 5 Vols.,	1862-71.
London,	*Messenger of Mathematics*,	1872.
	Mathematical Repository, The	1799-1804.
	Mathematical, Geometrical, and Philosophical Delights,	1792-98.
London,	*Miscellana Mathematica*, (Hutton's) 1 Vol.,	1775.
Paris,	*Nouvelles Annales de Mathematique*,	1842.
Aluwick,	*Northumbrian Mirror, The* 3 Vols.,	1837-40.
Dublin,	*Proceedings of the Royal Irish Academy*,	
	Philosophical Repository,	1801-4.
London,	*Philosophical Magazine*,	1798.
London,	*Quarterly Journal of Pure and Applied Mathematics*,	1857.
	Quarterly Visitor,	1814-15.
Holbeach,	*Scientific Receptacle*,	1825.
Bolton,	*Scientific Mirror*, 2 Nos.,	1829-30.
Dublin,	*Transactions of the Dublin Philosophical Society.*	
Edinburg,	*Transactions of the Royal Society of Edinburg*,	1872.
London,	*Transactions of the Royal Society*,	1665.
Leipzig,	*Zeitschrift fur Mathematik und Physik*,	1854.

REMARKS ON DIVISION.

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A large majority of the arithmetics in use in the United States teach that a "concrete number" can be divided by an abstract or pure number; that if the dividend is \$10 and the divisor 2, the quotient is \$5. Why this has been allowed to go unchallenged for so many generations is a psychological rather than a mathematical problem. Mathematicians have been neither scarce nor idle, but they seem to have been working upward and outward among the branches instead of digging down to the rootlets of the infinite tree of mathematical truth.

In the realm of number all the human mind can do is unite and take apart, involve and evolve, compose and analyze. Everthing is based upon addition. The inverse of additon is subtraction. Multiplication is a mere process of adding, hence *its* inverse is subtraction. If a given product is \$20 and the multiplicand \$5, we can find the multiplier by subtracting \$5 from \$20 until nothing remains. The number of times we subtract is 4, the multiplier. One number can be taken from another just as often as it is contained therein; hence, division is equivalent to subtraction, and is the inverse of multiplication.

If a given product is \$20, and the multiplier 4, we cannot by mere subtraction find the multiplicand. That is to say, multiplication has but one in-